STA 111 Practice Problems

1 Shooting Darts

I am playing darts. Suppose my probability of hitting the target is $p = 0.20$ and the throws are independent. For each question, write out an expression for the probability and identify the appropriate random variable including parameters. For 1.5 find a numeric answer.

1.1. What is the probability that in ten throws I don't hit the target every time?

1.2. What is the probability I hit the target exactly four times in ten throws?

1.3. What is the probability my first hit is on the fifth throw?

1.4. What is the probability that my third hit is on the eighth throw?

1.5. What is the approximate probability I hit the target 1,950 or fewer times in 10,000 throws?

2 Typos

Suppose the number of typos I make while typing has a Poisson distribution with mean $\lambda = 3.4$ typos per page.

2.1. What is the probability I make at least one typo on a page?

2.2. What is the probability I make exactly two typos on a page?

3 Free Throws

I am attempting free throws. Suppose my free throw success rate is $p = 0.45$ and my shots are independent.

3.1. What is the probability I make five free throws in ten attempts?

3.2. What is the probability my fifth basket is on my eleventh attempt?

3.3. What is the probability my first basket is on my third attempt?

3.4. What is the approximate probability I make 200 or fewer free throws in 500 attempts?

4 Backgammon

My roommate and I are playing backgammon repeatedly. Suppose the probability I win a game is 0.45 and the games are independent.

4.1. What is the probability I win three games in a six game series?

4.2. What is the probability my first win is on our third game?

4.3. What is the probability my third win is on our tenth game?

4.4. What is the probability my first win is on an odd-numbered game $(1, 3, 5, \ldots)$?

5 Die, Bart, Die

Bart rolls a fair die (labeled 1, 2, 3, 4, 5, 6) ten times. Answer the questions below and show your work.

5.1. What is the probability of rolling a 3 or higher on all of the ten rolls?

5.2. What is the probability of rolling a 3 or higher exactly four times in the ten rolls?

5.3. What is the probability of rolling a 3 or higher on at least one of the rolls?

6 Car Collisions

Suppose the number of weekly accidents caused by electric scooters at a Durham intersection is Poisson with mean 2.2.

6.1. What is the probability that there are no accidents in the next week?

6.2. What is the probability that there is at least one accident in the next week?

6.3. What is the probability that there is more than one but less than six accidents in the next week?

7 Soccer Games

On average, the Duke men's soccer team makes 1.5 goals per game.

7.1. What is the probability the Duke men's soccer team doesn't score any goals in their next game?

7.2. What is the probability they will score exactly three goals?

7.3. What is the probability they will score at least two goals?

8 Trivia Tournament

Ken Jennings is playing *Jeopardy!* head-to-head with James Holzhauer. Suppose the probability that Ken wins a game is 0.65 and the games are independent.

8.1. What is the probability that Ken wins exactly four games in a seven game series?

8.2. What is the probability that Ken wins at least one game in a seven game series?

8.3. What is the probability that Ken's first win is on the fourth game?

8.4. What is the probability that Ken's third win is on the fifth game?

8.5. What is the probability Ken's first win is on an even game (2, 4, 6, 8, ...)?

8.6. What is the probability Ken's first win is on an odd game (1, 3, 5, 7, ...)?

9 Phone Calls

Suppose the number of phone calls received by Duke OIT in an hour follows a Poisson distribution with mean 7.2.

9.1. What is the probability of no calls in an hour?

9.2. What is the probability of exactly five calls in an hour?

9.3. What is the probability of two or more calls received in an hour?

10 Weighted Die

A weighted die has the probability mass function given below.

x	f(x)
1	0.30
2	0.25
3	0.20
4	0.10
5	0.10
6	0.05

10.1. What is $\mathbb{E}[X]$?

10.2. What is $\mathbb{E}[X^2]$?

10.3. What is $\text{Var}(X)$?

10.4. What is $\text{SD}(X)$?

Now define a new random variable $Y = -5 \times X + 2$.

10.5. What is $\mathbb{E}[Y]$?

10.6. What is $\text{Var}(Y)$?

10.7. What is $\text{SD}(Y)$?

11 Dice Games

Roll the weighted die from Problem 10 three times and take the minimum of the face values[1]. If the minimum of the values is 1 you lose $1, if the minimum of the values is 2, 3, 4, or 5 you lose $0.50, and if the minimum of the values is 6 you win $5,000.

11.1. Denote by X the amount you gain or lose. What is the probability mass function of X?

11.2. What is $\mathbb{E}[X]$?

11.3. What is $\text{Var}(X)$?

12 Carnival Games

An urn contains thirteen balls. Ten are labeled "$0", two are labeled "$1", and one is labeled "$5". You draw three balls with replacement. Let X denote the maximum of the numbers on the three balls.

12.1. What is the probability distribution of X?

12.2. What is $\mathbb{E}$[X]?

12.3. What is VAR[X]?

12.4. What is SD[X]?

12.5. What are you willing to pay per game if you win $\$X$ on each of your draws from the urn?

13 Density Question

Suppose X has the probability density function given below.

$$f(x) = Cx^2 \text{ for } 0 < x < 9$$

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13.1. What is the value C that makes this a valid probability density function?

13.2. What is $\mathbb{E}[X]$?

13.3. What is $F(x) = \mathbb{P}(X \leq x)$?

13.4. What is $\mathbb{P}(X > 4)$?

13.5. What is the median of the distribution?

[1]So if you roll {1, 6, 4} the minimum is 1. If you roll {5, 6, 6} the minimum is 5

14 Lightbulb Lifetimes

Suppose the lifetime of a type of incandescent light bulb is exponentially distributed with mean 1,200 hours.

14.1. What is the probability a randomly selected light bulb lasts longer than 2,000 hours?

14.2. What is the median lifetime of this type of incandescent bulb?

14.3. What is the lifetime of a bulb that is in the worst 5% of lifetimes?

14.4. Light bulbs are rated failures if the lifetime is less than 200 hours. What is the probability that in a sample of twelve light bulbs, four or more are rated failures?

15 Tenting for Tickets

The proportion of Duke first-year students who are going to tent for tickets to the Duke vs U.N.C. basketball game is a random variable with the density below.

$$f(x) = K(x^3 - x^4), \qquad 0 \leq x \leq 1$$

15.1. Find the value of K that makes $f(x)$ a valid probability density function.

15.2. What is the probability that more than 80% of Duke first-years tent for tickets?

15.3. What is the expected proportion of Duke first-years who are going to tent for tickets?

15.4. What is the variance in the proportion of Duke first-years who are going to tent for tickets?

16 Properties of Covariance

Use the definition of covariance to show that the following properties are true. It may be helpful here to recall the properties of expectation from earlier in the semester.

16.1. $\text{Cov}(X, X) = \text{Var}(X)$.

16.2. $\text{Cov}(X, Y) = \text{Cov}(Y, X)$.

16.3. $\text{Cov}(X, a) = 0$.

16.4. $\text{Corr}(aX + b, cY + d) = \text{Corr}(X, Y)$, for a, $c > 0$.

17 Density Function

Suppose $f(x)$ has the density function below for $0 \leq x \leq 1$.

$$f(x) = Kx(3 - x)$$

17.1. What is the value of K that makes $f(x)$ a valid probability density function?

17.2. Find $F(x)$.

17.3. What is $\mathbb{P}(X > 0.5)$?

18 Probability Density Problems

Suppose X is a continuous random variable with probability density function given below.

$$f(x) = \begin{cases} K\sqrt{x} & \text{if } 0 < x < 9 \\ 0, & \text{otherwise} \end{cases}$$

18.1. State the two conditions necessary for $f(x)$ to be a valid probability density function.

(18.1.1)

(18.1.2)

18.2. Find the value of K so that $f(x)$ is a valid probability density function.

18.3. Find the cumulative distribution function $F(x)$.

18.4. Find the $\mathbb{P}(X > 3)$.

18.5. Find the $\mathbb{P}(4 \leq X \leq 6)$.

18.6. Find $\mathbb{E}[X]$.

18.7. Find the value c such that $\Pr(X \leq c) = 0.25$.

18.8. Find $\text{Var}[X]$.

19 Joint Density

Let $f(x, y)$ have the joint probability density function given below.

$$f(x,y) = \begin{cases} \frac{3}{4}x & \text{if } 0 < x < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

19.1. Confirm this is a valid joint probability density function.

19.2. What is the marginal density of X?

19.3. What is the marginal density of Y?

19.4. Are X and Y independent? Explain briefly.

20 Discrete Problem

A grocery store has self-checkout and regular check-out lines. Denote by X the number self-checkout lines in use at a certain time and by Y the number of regular check-out lines in use at the same time. The joint probability mass function of X and Y is shown below.

		y				
		0	1	2	3	
x	0	0.04	0.09	0.10	0.14	
	1	0.10	0.16	0.17	0.20	

20.1. What is the probability that no check out lines are in use?

20.2. What is the probability that at least one check out line is in use?

20.3. What is the probability that exactly one check out line is in use?

20.4. What is $\mathbb{P}(X \leq 1 \text{ and } Y \leq 1)$?

20.5. What is $\mathbb{P}(X = 1 \text{ or } Y \geq 2)$?

20.6. What is $\mathbb{P}(X = 0|Y = 2)$?

20.7. What is $\mathbb{E}[XY]$?

20.8. What is the marginal probability mass function of X?

20.9. What is $\mathbb{E}[X]$?

20.10. What is $\mathbb{P}(X \leq 1)$?

21 Density Question

Suppose X has the probability density function given below.

$$f(x) = \frac{x^2}{24} \quad \text{for } -2 \leq x \leq 4$$

21.1. What is $\mathbb{E}[X]$?

21.2. Write out a single integral expression for $\text{Var}(X)$. No need to calculate.

21.3. What is $F(x) = \mathbb{P}(X \leq x)$?

21.4. What is the median of the distribution?

22 Expectation & Variance

Suppose $\mathbb{E}[X] = -2$ and $\text{Var}(X) = 6$. Define $Y = -2 \times X + 6$, $Z = X^2$ and $W = -3 \times X - 2$.

22.1. What is $\mathbb{E}[Y]$?

22.2. What is $\text{Var}(Y)$?

22.3. What is $\text{SD}(Y)$?

22.4. What is $\mathbb{E}[Z]$?

22.5. What is $\mathbb{E}[W]$?

22.6. What is $\text{Var}(W)$?

23 Expectation

Suppose $\mathbb{E}(X) = 3$ and $SD(X) = 4$. Let $Y = -5 \times X + 15$ and $Z = 2 \times X - 4$.

23.1. What is $\mathbb{E}(Y)$?

23.2. What is $SD(Y)$?

23.3. What is $\mathbb{E}(Z)$?

23.4. What is $VAR(Z)$?