

# STA 111 Practice Problems

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## 1 Adult Male Heights

Suppose a random sample of 100 American adult male heights has a sample mean of 68.7 inches and the *known* standard deviation is 3.8 inches.

- 1.1. Construct a 95% confidence interval for the average height of American adult men.

For the three situations below, state how the confidence interval will change compared to your answer in 1.1. In each situation, we only make the specified change and we still obtain a sample mean of 68.7 with known standard deviation 3.8 inches. Provide a brief justification for each answer.

- 1.2. We construct a 99% confidence interval instead of a 95% confidence interval.
- 1.3. We construct a 90% confidence interval instead of a 95% confidence interval.
- 1.4. The sample size is 50 instead of 100.
- 1.5. The sample size is 500 instead of 100.

## 2 2020 Presidential Election

NBC/Marist recently randomly sampled 1,733 registered North Carolina voters and asked “If November’s election for president were held today, whom would you support if the candidates are Bernie Sanders and Donald Trump.” 51.07% answered Bernie Sanders.

- 2.1. Construct a 90% confidence interval on the true proportion of North Carolina voters who are planning to vote for Sanders and provide a precise one-sentence interpretation of your interval in the context of the problem.

## 3 Coronavirus Quarantine

The research firm Ipsos recently sampled 1,005 American adults and asked “To what extent would you support or oppose the United States placing a quarantine on a major city, such as New York, Chicago or San Francisco, if [coronavirus] is detected there?” A 95% confidence interval on the proportion who would support a quarantine is (0.7736, 0.8240). For each of the following changes, state whether the width of the interval will INCREASE, DECREASE, or STAY THE SAME. Consider only the specified change and justify each answer in a brief sentence.

- 3.1. The sample size is halved.
- 3.2. The confidence level is decreased from 95% to 90%.
- 3.3. Instead of 80% reporting they would support a quarantine, only 60% report they would support a quarantine.

## 4 Bloomberg Blues

Economist / YouGov recently (2-23 - 2-25) took a random sample of 1,493 American adult citizens and found that 24% of them have a favorable opinion of Mike Bloomberg.

- 4.1. Construct a 99% upper confidence bound on the true proportion of American adult citizens who have a favorable opinion of Mike Bloomberg and provide a precise one-sentence interpretation of your interval in the context of the problem.

## 5 Birth Weights

Denote by  $X$  the birth weight in pounds of children born in North Carolina and assume  $X \sim N(\mu, \sigma)$ . A random sample of  $n = 14$  is taken, yielding the weights below.

7.31, 5.75, 8.88, 3.81, 6.81, 7.50, 7.69, 8.38, 8.38, 8.06, 9.94, 7.31, 7.75, 8.06

- 5.1. Suppose  $\sigma^2 = 2.25$ . What is a 90% confidence interval for  $\mu$ ?
- 5.2. Suppose  $\sigma^2 = 2.25$ . What is a 90% upper confidence interval for  $\mu$ ?
- 5.3. Suppose  $\sigma^2$  is unknown. What is a 90% confidence interval for  $\mu$ ?

## 6 Bootstrap

You are interested in constructing a 95% confidence interval for the median IQ of American undergraduate students. You take a random sample of  $n = 61$  students and find a median IQ of 103.5. You resample 1,000 times from the original sample data with replacement using a sample of size 61 and calculate the median for each resample. The standard deviation of all of the bootstrap medians is 1.5598. Summary data is provided below.

	0%	2.5%	5%	10%	90%	95%	97.5%	100%
	99.0	102.0	102.0	102.0	106.5	107.0	107.5	109.0

- 6.1. Construct a 95% bootstrap percentile confidence interval.
- 6.2. Construct a 95% bootstrap standard error interval.

## 7 Trump Approval

A recent Economist / YouGov poll conducted between October 6 and October 8 randomly sampled 1,500 American adult citizens and asked “Do you approve or disapprove of the way Donald Trump is handling his job as President?” 47% responded approve and 53% responded disapprove.

- 7.1. Construct a 99% confidence interval for the proportion of all U.S. adult citizens who approve of the way President Trump is handling his job as President.
- 7.2. Interpret your interval in a brief sentence.
- 7.3. What  $n$  would be required for the width of your 99% confidence interval to be at most 0.05 for any  $\hat{p}$ ?

## 8 Sample Size

- 8.1. Interested in investigating public opinions on coronavirus in more depth, you would like to construct a 99% confidence interval on the true proportion of American adults who believe that gatherings of more than 1,000 people should be cancelled. If you wish to limit your margin of error to 3%, how many individuals should you sample?

## 9 Corona Polling

An Economist / YouGov poll conducted between March 1 and March 3 2020 asked a random sample of 1,483 adult Americans, “Do you think the authorities should require people who can work from home to work from home?” 36% responded “Yes.”

- 9.1. Construct a 90% confidence interval on the true proportion of adult Americans who think the authorities should require people who can work from home to work from home.
- 9.2. Provide a concise, one-sentence interpretation of your interval in the context of the problem.

## 10 Cursed Confidence Intervals

In a survey of 1500 adult Americans, 30% responded that they believe some people can put curses on other people.

- 10.1. Construct a 95% confidence upper bound on the true proportion of adult Americans who believe some people can put curses on other people.
- 10.2. Provide a one-sentence interpretation of your upper bound in the context of the problem.
- 10.3. You wish to construct a 95% confidence interval on the true proportion of American adults who believe in ghosts. If you want the margin of error to be limited to 2% for any  $\hat{p}$ , how many Americans should you sample?

## 11 $t$ Intervals

Find the  $t$  critical value for a two-sided confidence interval for each of the situations below.

- |                                            |                                            |
|--------------------------------------------|--------------------------------------------|
| 11.1. $C = 0.95$ , degrees of freedom = 10 | 11.4. $C = 0.99$ , degrees of freedom = 5  |
| 11.2. $C = 0.95$ , degrees of freedom = 15 | 11.5. $C = 0.98$ , degrees of freedom = 24 |
| 11.3. $C = 0.99$ , degrees of freedom = 15 | 11.6. $C = 0.95$ , degrees of freedom = 40 |

## 12 $t$ Intervals

Find the  $t$  critical value for a one-sided confidence lower or upper bound for each of the situations below.

- |                                            |                                             |
|--------------------------------------------|---------------------------------------------|
| 12.1. $C = 0.95$ , degrees of freedom = 10 | 12.4. $C = 0.99$ , degrees of freedom = 5   |
| 12.2. $C = 0.95$ , degrees of freedom = 15 | 12.5. $C = 0.975$ , degrees of freedom = 24 |
| 12.3. $C = 0.99$ , degrees of freedom = 15 | 12.6. $C = 0.95$ , degrees of freedom = 40  |

## 13 Confidence Interval

Determine the confidence level  $C$  for the following large-sample confidence bounds / intervals. Note that no work is required here.

- 13.1.  $\bar{x} + 1.28 \times \text{SE}$
- 13.2.  $\bar{x} - 1.64 \times \text{SE}$
- 13.3.  $\bar{x} \pm 1.28 \times \text{SE}$

## 14 Diamond Joe

The polling firm Quinnipiac randomly sampled 1,195 registered voters between October 11 and October 13 and found that 41% have a favorable opinion of Joe Biden.

- 14.1. What is a 95% upper confidence bound for the true proportion of registered voters who have a favorable opinion of Joe Biden?

## 15 Fake News

According to a recent survey ( $n = 5,107$ ) conducted July 8 - 21 by the Pew Research Center, a 95% confidence interval for the proportion of U.S. adults who believe social media companies have too much control over the mix of news that people see is  $L = 60.7\%$ ,  $U = 63.3\%$ .

- 15.1. What is the point estimate?

15.2. What is the margin of error?

15.3. What is the standard error?

Identify the following statements as TRUE or FALSE and provide a concise, one-sentence justification for each.

15.4. In order to halve the margin of error, we would need to double the sample size.

15.5. There is a 95% chance that the true population proportion is between 0.607 and 0.633.

15.6. If we considered many random samples of U.S. adults of size  $n = 5,107$  and calculated 95% confidence intervals for each, about 95% of them will include the true population proportion.

15.7. If we considered many random samples of U.S. adults of size  $n = 5,107$  and calculated the sample proportion for each, about 95% of the sample proportions will be between 0.607 and 0.633.

15.8. We are 95% confident that between 60.7% and 63.3% of Americans in this sample believe social media companies have too much control over the mix of news people see.

## 16 Video Games

You are concerned that STA 111 students are spending too much time playing video games and wish to construct a 95% upper confidence bound on the average amount of time students spend playing video games each week. You draw a sample of 15 students from our class of 100 students and find the average is 6 hours with a sample standard deviation of 45 minutes.

16.1. What is the upper confidence bound?

16.2. Provide a brief, one-sentence interpretation in the context of the problem.