

STA 111 Practice Problems

1 Basketballs

You order a large shipment of basketballs to practice for the upcoming NCAA Basketball season. Unfortunately, some of the basketballs are defective. Suppose the proportion of defective basketballs is p and the prior distribution on p is $\text{Beta}(\alpha = 2, \beta = 20)$. You randomly select 50 basketballs at random and find that 2 are defective.

- 1.1. What is the posterior distribution of p ?
- 1.2. What is the estimate of p that minimizes the squared error?

2 Dog Food

You order a large shipment of canned dog food, but unfortunately some of the cans are spoiled. Suppose the proportion of spoiled cans is p and the prior distribution on p is $\text{Beta}(\alpha = 2, \beta = 50)$. You randomly select 100 cans at random and find that two are spoiled.

- 2.1. What is the posterior distribution of p ?
- 2.2. What is the estimate of p that minimizes the squared error?

3 Lecture Mistakes

Suppose the number of algebra errors I make per hour of lecture has a Poisson distribution with unknown parameter θ . Based on your knowledge of my teaching in previous courses you believe that the distribution of θ is Gamma with mean $2/3$ and variance $2/9$. I lecture for four hours and make five mistakes.

- 3.1. What is the posterior distribution for θ ?
- 3.2. What is the Bayes estimator of θ that minimizes the absolute error?

4 Normal Bayes

Suppose a random sample of size $n = 20$ is taken from a normal distribution with the mean θ is unknown and the known variance σ^2 is 1. Suppose $\bar{x} = 10$, and that the posterior distribution of θ is normal with mean 8 and variance $1/25$. What was the prior distribution of θ ?

5 Type Racer

Suppose the number of typing errors I make per page of text has a Poisson distribution with unknown parameter θ , and that the prior distribution of θ is gamma with $\alpha = 2$ and $\beta = 4$. I type for four pages and make five mistakes. You can visualize the Gamma distribution at <https://homepage.divms.uiowa.edu/~mbognar/applets/gamma.html>.

- 5.1. What is the posterior distribution for θ ?
- 5.2. What is the Bayes estimator of θ that minimizes the square error?
- 5.3. What is the Bayes estimator of θ that minimizes the absolute error? *Hint: Just using words is fine here.*

6 Duke Heights

Suppose heights of male Duke students are normally distributed with unknown mean μ and known standard deviation 2.5 inches. The prior distribution of μ is normal with mean 69 inches and standard deviation 1.75 inches. Ten men are selected at random from the Duke population and their average height is found to be 70 inches.

- 6.1. What is the posterior distribution of μ ?
- 6.2. What is the estimate of μ that minimizes the squared error?
- 6.3. What is the estimate of μ that minimizes the absolute error?